Constraints in analysis of synchronous programs

Synchron' 2006

Christophe Mauras
Christophe.Mauras@univ-nantes.fr

LINA/Université de Nantes

Overview

– References : some works that had influenced mine.
– Experiments : those that work and others
– Tutorial : how to exploit constraints, to reformulate known techniques in analysis of synchronous programs.

References

- 20 years ago :
  - W. Buttner and H. Simonis, Embedding boolean expressions into logic programming
  - M.M. Corsini and A. Rauzy, Symbolic model-checking and constraint logic programming : a cross-fertilisation
- 10 years ago :
  - G. Delzanno and A. Podelski, Model Checking in CLP
  - W. Chan and R. Anderson and P. Beame and D. Notkin, Combining constraints solving and symbolic model-checking for a class of systems with non-linear constraints
  - F. Benhamou and W. Older, Applying interval arithmetic to real, integer and boolean constraints

References

- Recent works : 1st workshop on Constraints in Software Testing, Verification and Analysis (CSTVA'06)
  - The V3F Project, B.Blanc, Fabrice Bouquet, Arnaud Gotlieb, Bertrand Jeannet, Thierry Jeron, Bruno Legeard, Bruno Marre, Claude Michel, Michel Rueher
  - Generating random values using Binary Decision Diagrams and Convex Polyhedra Erwan Jahier, Pascal Raymond
  - ...

Experiments

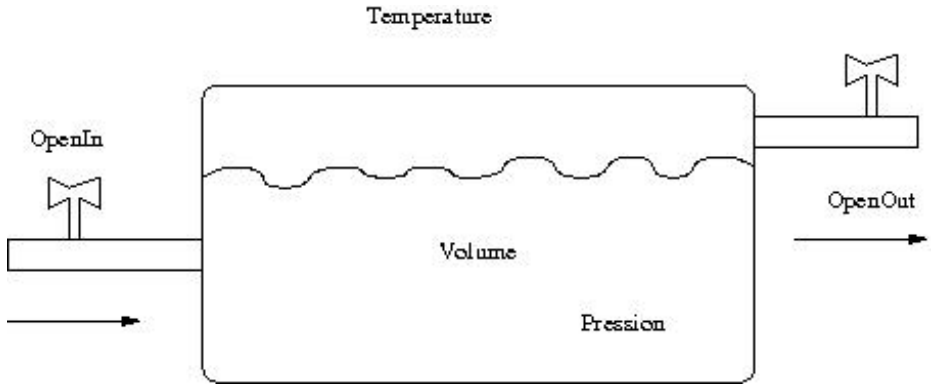
- Symbolic execution on interpreted automata (YoYo, 1996)
  - Simulation, verification, invariant synthesis
  - Description language : very poor
  - Execution model : only forward, no fixpoint
- Timed Default Concurrent Constraints
  - A language unifying concepts of synchronous languages
  - A prototype interpreter restricted to closed programs
  - A tentative to patch it for verification purposes (aborted)
- Deductive Model Checking
  - A method to compute set of states from a transition system encoded as constrained facts
  - A tentative to re-use it for TDCC (aborted)

Tutorial

5 Good reasons to use constraints :

– Ability to specify and execute incomplete designs
– Unicity of formalism for system under test, its environment and properties to be verified
– Bidirectionnality : pre and post are computed in a similar way
– Ability to represent set of states (characteristic function) or set of trajectories
– Availability of many solvers.

A Controller for a steam boiler



A transition system

Translated in CLP(B)+CLP(Q) from Lustre

```
post(Openin,Openout,Pressure,Volume,Nopenin,Nopenout,Npressure,Nvolume):-
  (sat(Nopenout), {Npressure-6>=0}, sat(~Nopenin), {-Nvolume+19>=0}, {-Pressure+Npressure=0}, {Temp-11>=0}, {-Volume>=0}, {-Volume+Nvolume=0}, sat(~Ok), sat(Alarm), sat(~Openout), sat(~Openin));
  (sat(Nopenout), {Npressure-6>=0}, sat(~Nopenin), {-Nvolume+19>=0}, {-Pressure+Npressure-1=0}, {Temp-11>=0}, {Volume-1>=0}, {-Volume+Nvolume+5=0}, sat(~Ok), sat(Alarm), sat(~Openout), sat(~Ope
  (sat(Nopenout), {-Nvolume+19>=0}, sat(~Nopenin), {Npressure-6>=0}, {-Pressure+Npressure=0}, {Temp-11>=0}, {-Volume>=0}, {-Volume+Nvolume=0}, sat(Ok), sat(~Openout), sat(~Alarm), sat(Openin));
  (sat(Nopenout), {-Nvolume+19>=0}, sat(~Nopenin), {Npressure-6>=0}, {-Pressure+Npressure=0}, {Temp-11>=0}, {-Volume>=0}, {-Volume+Nvolume=0}, sat(~Ok), sat(Openin), sat(~Alarm), sat(Openout));
  (sat(Nopenout), {-Nvolume+19>=0}, sat(~Nopenin), {Npressure-6>=0}, {-Pressure+Npressure-1=0}, {Temp-11>=0}, {Volume-1>=0}, {-Volume+Nvolume+5=0}, sat(Ok), sat(~Openout), sat(~Alarm), sat(Open
  (sat(Nopenout), {-Nvolume+19>=0}, sat(~Nopenin), {Npressure-6>=0}, {-Pressure+Npressure-1=0}, {Temp-11>=0}, {Volume-1>=0}, {-Volume+Nvolume+5=0}, sat(~Ok), sat(Openin), sat(~Alarm), sat(Openc
  (sat(Nopenout), {-Nvolume+19>=0}, sat(Nopenin), {-Npressure+5>=0}, {-Pressure+Npressure=0}, {Temp-11>=0}, {Volume>=0}, {-Volume+Nvolume=0}, sat(~Ok), sat(Openin), sat(~Alarm), sat(Openout));
  (sat(Nopenout), {-Nvolume+19>=0}, sat(Nopenin), {-Npressure+5>=0}, {-Pressure+Npressure-1=0}, {Temp-11>=0}, {Volume-1>=0}, {-Volume+Nvolume+5=0}, sat(~Ok), sat(Openin), sat(~Alarm), sat(Openc
  (sat(Nopenout), sat(~Nopenin), {-Nvolume+19>=0}, {-Pressure+Npressure=0}, {Temp-11>=0}, {-Volume>=0}, {-Volume+Nvolume=0}, sat(Ok), sat(~Openin), sat(~Alarm), sat(Openout));
  (sat(Nopenout), sat(~Nopenin), {-Nvolume+19>=0}, {-Pressure+Npressure-1=0}, {Temp-11>=0}, {Volume-1>=0}, {-Volume+Nvolume+5=0}, sat(Ok), sat(~Openin), sat(~Alarm), sat(Openout));
  (sat(Nopenout), sat(Nopenin), {-Pressure+Npressure=0}, {-Temp+10>=0}, {-Pressure>=0}, {-Volume+Nvolume=0}, sat(~Ok), sat(Alarm), sat(~Openout), sat(~Openin));
  (sat(Nopenout), sat(Nopenin), {-Pressure+Npressure+1=0}, {-Temp+10>=0}, {Pressure-1>=0}, {-Volume+Nvolume-5=0}, sat(~Ok), sat(Alarm), sat(~Openout), sat(~Openin));
  (sat(Nopenout), sat(~Nopenin), {-Pressure+Npressure=0}, {-Temp+10>=0}, {-Pressure>=0}, {-Volume+Nvolume=0}, sat(Ok), sat(~Openin), sat(~Alarm), sat(Openout));
  (sat(Nopenout), sat(~Nopenin), {-Pressure+Npressure+1=0}, {-Temp+10>=0}, {Pressure-1>=0}, {-Volume+Nvolume-5=0}, sat(Ok), sat(~Openin), sat(~Alarm), sat(Openout));
  (sat(~Nopenout), sat(Nopenin), {-Pressure+Npressure=0}, {-Temp+10>=0}, {-Pressure>=0}, {-Volume+Nvolume=0}, sat(Ok), sat(~Openout), sat(~Alarm), sat(Openin));
  (sat(~Nopenout), sat(Nopenin), {-Pressure+Npressure+1=0}, {-Temp+10>=0}, {Pressure-1>=0}, {-Volume+Nvolume-5=0}, sat(Ok), sat(~Openout), sat(~Alarm), sat(Openin));
  (sat(~Nopenout), sat(~Nopenin), {-Pressure+Npressure=0}, {-Temp+10>=0}, {-Pressure>=0}, {-Volume+Nvolume=0}, sat(~Ok), sat(Openin), sat(~Alarm), sat(Openout));
  (sat(~Nopenout), sat(~Nopenin), {-Pressure+Npressure+1=0}, {-Temp+10>=0}, {Pressure-1>=0}, {-Volume+Nvolume-5=0}, sat(~Ok), sat(Openin), sat(~Alarm), sat(Openout));
  (sat(~Nopenout), sat(Nopenin), {-Npressure+5>=0}, {Nvolume-20>=0}, {-Pressure+Npressure=0}, {Temp-11>=0}, {-Volume>=0}, {-Volume+Nvolume=0}, sat(~Ok), sat(Alarm), sat(~Openout), sat(~Openin));
  (sat(~Nopenout), sat(Nopenin), {-Npressure+5>=0}, {Nvolume-20>=0}, {-Pressure+Npressure-1=0}, {Temp-11>=0}, {Volume-1>=0}, {-Volume+Nvolume+5=0}, sat(~Ok), sat(Alarm), sat(~Openout), sat(~Ope
  (sat(~Nopenout), sat(Nopenin), {-Npressure+5>=0}, {-Pressure+Npressure=0}, {Temp-11>=0}, {-Volume>=0}, {-Volume+Nvolume=0}, sat(Ok), sat(~Openout), sat(~Alarm), sat(Openin));
  (sat(~Nopenout), sat(Nopenin), {-Npressure+5>=0}, {-Pressure+Npressure-1=0}, {Temp-11>=0}, {Volume-1>=0}, {-Volume+Nvolume+5=0}, sat(Ok), sat(~Openout), sat(~Alarm), sat(Openin));
  (sat(~Nopenout), {-Npressure+5>=0}, sat(~Nopenin), {-Nvolume+19>=0}, {-Pressure+Npressure=0}, {Temp-11>=0}, {-Volume>=0}, {-Volume+Nvolume=0}, sat(~Ok), sat(Alarm), sat(~Openout), sat(~Openin
  (sat(~Nopenout), {-Npressure+5>=0}, sat(~Nopenin), {-Nvolume+19>=0}, {-Pressure+Npressure-1=0}, {Temp-11>=0}, {Volume-1>=0}, {-Volume+Nvolume+5=0}, sat(~Ok), sat(Alarm), sat(~Openout), sat(~O
  (sat(~Nopenout), sat(Nopenin), {-Npressure+5>=0}, {Nvolume-20>=0}, {-Pressure+Npressure=0}, {Temp-11>=0}, {-Volume>=0}, {-Volume+Nvolume=0}, sat(Ok), sat(~Openin), sat(~Alarm), sat(Openout));
  (sat(~Nopenout), sat(Nopenin), {-Npressure+5>=0}, {Nvolume-20>=0}, {-Pressure+Npressure=0}, {Temp-11>=0}, {-Volume>=0}, {-Volume+Nvolume=0}, sat(~Ok), sat(Openin), sat(~Alarm), sat(Openout));
  (sat(~Nopenout), sat(Nopenin), {-Npressure+5>=0}, {Nvolume-20>=0}, {-Pressure+Npressure-1=0}, {Temp-11>=0}, {Volume-1>=0}, {-Volume+Nvolume+5=0}, sat(Ok), sat(~Openin), sat(~Alarm), sat(Openc
  (sat(~Nopenout), sat(Nopenin), {-Npressure+5>=0}, {Nvolume-20>=0}, {-Pressure+Npressure-1=0}, {Temp-11>=0}, {Volume-1>=0}, {-Volume+Nvolume+5=0}, sat(~Ok), sat(Openin), sat(~Alarm), sat(Openc
  (sat(~Nopenout), {Nvolume-20>=0}, sat(~Nopenin), {Npressure-6>=0}, {-Pressure+Npressure=0}, {Temp-11>=0}, {-Volume>=0}, {-Volume+Nvolume=0}, sat(Ok), sat(~Openin), sat(~Alarm), sat(Openout));
  (sat(~Nopenout), {Nvolume-20>=0}, sat(~Nopenin), {Npressure-6>=0}, {-Pressure+Npressure=0}, {Temp-11>=0}, {-Volume>=0}, {-Volume+Nvolume=0}, sat(Ok), sat(~Openout), sat(~Alarm), sat(Openin));
  (sat(~Nopenout), {Nvolume-20>=0}, sat(~Nopenin), {Npressure-6>=0}, {-Pressure+Npressure=0}, {Temp-11>=0}, {-Volume>=0}, {-Volume+Nvolume=0}, sat(~Ok), sat(Alarm), sat(~Openout), sat(~Openin));
  (sat(~Nopenout), {Nvolume-20>=0}, sat(~Nopenin), {Npressure-6>=0}, {-Pressure+Npressure=0}, {Temp-11>=0}, {-Volume>=0}, {-Volume+Nvolume=0}, sat(~Ok), sat(Openin), sat(~Alarm), sat(Openout));
  (sat(~Nopenout), {Nvolume-20>=0}, sat(~Nopenin), {Npressure-6>=0}, {-Pressure+Npressure-1=0}, {Temp-11>=0}, {Volume-1>=0}, {-Volume+Nvolume+5=0}, sat(Ok), sat(~Openin), sat(~Alarm), sat(Openc
  (sat(~Nopenout), {Nvolume-20>=0}, sat(~Nopenin), {Npressure-6>=0}, {-Pressure+Npressure-1=0}, {Temp-11>=0}, {Volume-1>=0}, {-Volume+Nvolume+5=0}, sat(Ok), sat(~Openout), sat(~Alarm), sat(Open
  (sat(~Nopenout), {Nvolume-20>=0}, sat(~Nopenin), {Npressure-6>=0}, {-Pressure+Npressure-1=0}, {Temp-11>=0}, {Volume-1>=0}, {-Volume+Nvolume+5=0}, sat(~Ok), sat(Alarm), sat(~Openout), sat(~Ope
  (sat(~Nopenout), {Nvolume-20>=0}, sat(~Nopenin), {Npressure-6>=0}, {-Pressure+Npressure-1=0}, {Temp-11>=0}, {Volume-1>=0}, {-Volume+Nvolume+5=0}, sat(~Ok), sat(Openin), sat(~Alarm), sat(Openc
```

Computing Post predicate transformer as constraint solving

```
post(In,Out,P,V,Nin,Nout,Np,Nv).
```

```
In = 0,Nin = 0,Out = 0,Nout = 1,{P=Np},{Nv=V},{V=<0},{Np>=6} ? ;
In = 0,Nin = 0,Out = 0,Nout = 1,{P= -1+Np},{V=5+Nv},{Nv=<19},{Np>=6},{Nv>= -4} ? ;
In = 1,Nin = 0,Out = 0,Nout = 1,{P=Np},{Nv=V},{V=<0},{Np>=6} ? ;
In = 1,Nin = 0,Out = 1,Nout = 1,{P=Np},{Nv=V},{V=<0},{Np>=6} ? ;
In = 1,Nin = 0,Out = 0,Nout = 1,{P= -1+Np},{V=5+Nv},{Nv=<19},{Nv>= -4},{Np>=6} ? ;
In = 1,Nin = 0,Out = 1,Nout = 1,{P= -1+Np},{V=5+Nv},{Nv=<19},{Nv>= -4},{Np>=6} ? ;
In = 1,Nin = 1,Out = 1,Nout = 1,{P=Np},{Nv=V},{Np=<5},{V=<0} ?
...
```

Computing image of a defined state

```
post(0,1,7,10,Nopenin,Nopenout,Npressure,Nvolume).
```

```
Nopenin = 0,Nvolume = 5,Nopenout = 1,Npressure = 8 ? ;
Nopenin = 0,Nvolume = 15,Nopenout = 1,Npressure = 6 ? ;
```

Useful for simulation

Computing image of a set of states

```
{P>=5},{V=<12},
post(1,0,P,V,Nopenin,Nopenout,Npressure,Nvolume).
```

```
Nopenin = 0,Nopenout = 1,{Nvolume=V},{P=Npressure},{V=<0},{Npressure>=6} ? ;
Nopenin = 0,Nopenout = 1,{Nvolume= -5+V},{P= -1+Npressure},{V=<12},{Npressure>=6},{V>=1} ? ;
Nopenin = 1,Nopenout = 0,{Npressure= -1+P},{Nvolume=5+V},{V=<12},{P>=5} ? ;
P = 5,Nopenin = 1,Nopenout = 0,Npressure = 5,{Nvolume=V},{V=<0} ? ;
```

Useful for symbolic execution/observability in structural testing

Computing pre-image of a (set of) state(s)

```
post(Openin,Openout,Pressure,Volume,_,_,5,20).
```

```
Openin = 0,Volume = 15,Openout = 0,Pressure = 6 ? ;
Openin = 0,Volume = 15,Openout = 1,Pressure = 6 ? ;
Openin = 1,Volume = 15,Openout = 0,Pressure = 6 ? ;
Openin = 1,Volume = 15,Openout = 1,Pressure = 6 ? ;
Openin = 0,Volume = 25,Openout = 0,Pressure = 4 ? ;
Openin = 1,Volume = 25,Openout = 0,Pressure = 4 ? ;
Openin = 0,Volume = 25,Openout = 1,Pressure = 4 ? ;
Openin = 1,Volume = 25,Openout = 1,Pressure = 4 ? ;
```

Useful for debugging/controllability in structural testing

Reachability analysis

```
initial(Openin,Openout,Pressure,Volume):-
    sat(Openout), sat(~Openin), {Pressure-7=0}, {Volume-10=0}.
reachable(Openin,Openout,Pressure,Volume,T):-
    {T=0}, initial(Openin,Openout,Pressure,Volume).
reachable(Openin,Openout,Pressure,Volume,T):-
    {PT-T+1=0}, reachable(Popenin,Popenout,Ppressure,Pvolume,PT),
    post(Popenin,Popenout,Ppressure,Pvolume,Openin,Openout,Pressure,Volume).
```

Aim : to compute Post*(Init)

Walking along reachable states

```
reachable(Openin,Openout,Pressure,Volume,T).
```

```
T = 0,Openin = 0,Volume = 10,Openout = 1,Pressure = 7 ? ;
T = 1,Openin = 0,Volume = 5,Openout = 1,Pressure = 8 ? ;
T = 1,Openin = 0,Volume = 15,Openout = 1,Pressure = 6 ? ;
T = 2,Openin = 0,Volume = 0,Openout = 1,Pressure = 9 ? ;
T = 2,Openin = 0,Volume = 10,Openout = 1,Pressure = 7 ? ;
T = 2,Openin = 0,Volume = 10,Openout = 1,Pressure = 7 ? ;
T = 2,Openin = 0,Volume = 20,Openout = 1,Pressure = 5 ? ;
T = 3,Openin = 0,Volume = 0,Openout = 1,Pressure = 9 ? ;
...
```

Need for a more compact representation.

Time abstraction

```
reachable(Openin,Openout,Pressure,Volume,_).
```

```
Openin = 0,Volume = 10,Openout = 1,Pressure = 7 ? ;
Openin = 0,Volume = 5,Openout = 1,Pressure = 8 ? ;
Openin = 0,Volume = 15,Openout = 1,Pressure = 6 ? ;
Openin = 0,Volume = 0,Openout = 1,Pressure = 9 ? ;
Openin = 0,Volume = 10,Openout = 1,Pressure = 7 ? ;
Openin = 0,Volume = 10,Openout = 1,Pressure = 7 ? ;
Openin = 0,Volume = 20,Openout = 1,Pressure = 5 ? ;
Openin = 0,Volume = 0,Openout = 1,Pressure = 9 ?
...
```

Need for widening operator.

Backward analysis

Is a given state reachable?

```
reachable(Openin,Openout,3,30,T).
```

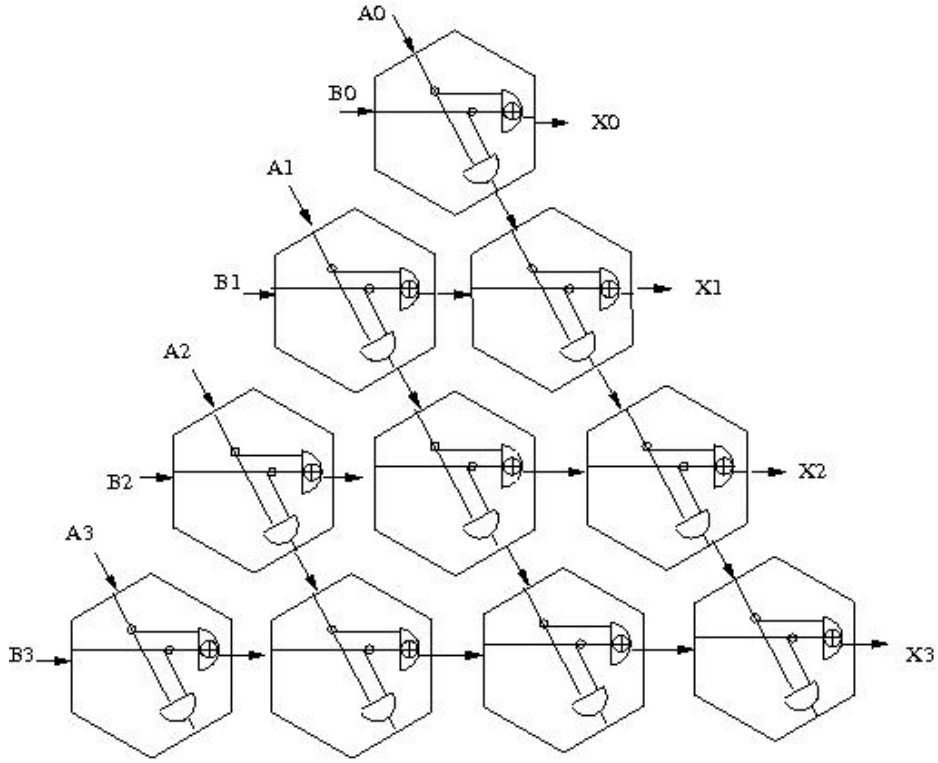
```
T = 4,Openin = 0,Openout = 1 ? ;
T = 6,Openin = 0,Openout = 1 ? ;
T = 6,Openin = 0,Openout = 1 ? ;
T = 6,Openin = 0,Openout = 1 ? ;
T = 6,Openin = 1,Openout = 0 ? ;
T = 6,Openin = 1,Openout = 0 ? ;
T = 6,Openin = 1,Openout = 0 ? ;
T = 8,Openin = 0,Openout = 1 ?
...
```

Property checking

```
{P>=8},reachable(_,_,P,V,T).

P = 8,T = 1,V = 5 ? ;
P = 9,T = 2,V = 0 ? ;
P = 9,T = 3,V = 0 ? ;
P = 8,T = 3,V = 5 ? ;
P = 8,T = 3,V = 5 ? ;
P = 8,T = 3,V = 5 ? ;
P = 9,T = 4,V = 0 ?
...
```

Diagonal carry adder



Equivalence checking in CLP(B)

```
andxor(A,B,X,C):-
    sat(X =:= A#B),sat(C =:= A*B).
adderandxor([A0,A1,A2,A3],[B0,B1,B2,B3],[X0,X1,X2,X3]):-
    andxor(A0,B0,X0,C00),
    andxor(A1,B1,X10,C10),andxor(X10,C00,X1,C11),
    andxor(A2,B2,X20,C20),andxor(X20,C10,X21,C21), andxor(X21,C11,X2,C22),
    andxor(A3,B3,X30,C30),andxor(X30,C20,X31,C31), andxor(X31,C21,X32,C32), andxor(X32,C22,X3,C33).

adder1bit(Ain,Bin,Cin,Xout,Cout) :-
    sat(Xout =:= Ain#Bin#Cin),sat(Cout =:= Ain*Bin+Ain*Cin+Bin*Cin).
adder4bits([A0,A1,A2,A3],[B0,B1,B2,B3],[X0,X1,X2,X3]) :-
    adder1bit(A0,B0,0,X0,C1),
    adder1bit(A1,B1,C1,X1,C2),
    adder1bit(A2,B2,C2,X2,C3),
    adder1bit(A3,B3,C3,X3,C4).

test(A0,A1,A2,A3,B0,B1,B2,B3) :-
    adder4bits([A0,A1,A2,A3],[B0,B1,B2,B3],[X0,X1,X2,X3]),
    adderandxor([A0,A1,A2,A3],[B0,B1,B2,B3],[Y0,Y1,Y2,Y3]),
    sat(X0=:=Y0),sat(X1=:=Y1),sat(X2=:=Y2),sat(X3=:=Y3).

| ?- test(A0,A1,A2,A3,B0,B1,B2,B3).
true
```

Conclusion

– Constraints programming is a good paradigm for analysis of synchronous programs
– A new solver is needed to represent globally set of states with boolean and numerical variables
– Concurrent constraints already need constraints for one execution path. What kind of constraints for a set of execution paths ?