

Residual for Component Specifications

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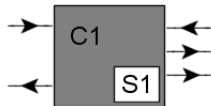
PhD Defense - December 18, 2007

Motivation

- Component-based design improves **readability** and **reliability**
- Crucial question : how to characterize reuse of a component ?
 - ▶ Reuse at the **signature** level:
⇒ Syntactic
 - vs.**
 - ▶ Reuse at the **behavioral** level.
⇒ Semantic

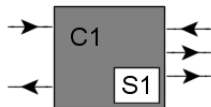
The quotient operation (1/2)

- Component = a pair $(\mathcal{C}, \mathcal{S})$ such that $\mathcal{C} \models \mathcal{S}$

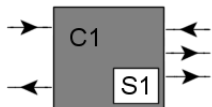


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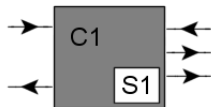


- Behavioral reuse :



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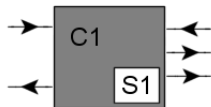


- Behavioral reuse :

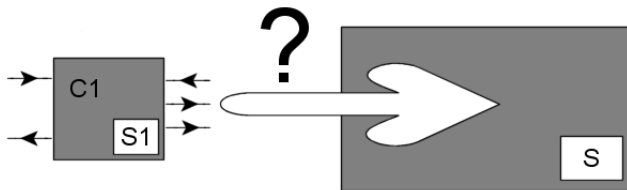


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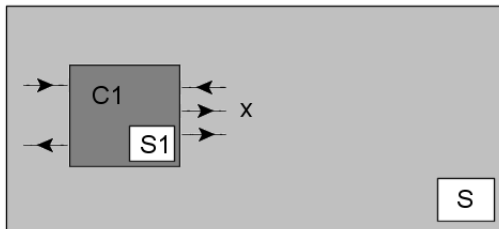
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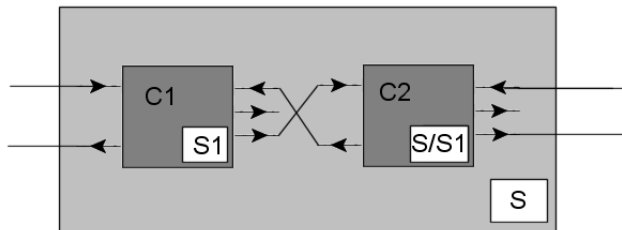
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The quotient operation (2/2)



The quotient operation (2/2)



Characteristic property :

$$C_2 \models S/S_1 \Leftrightarrow \forall C_1. [C_1 \models S_1 \Rightarrow C_1 \times C_2 \models S]$$

Related works

- Equation solving : $C \times X \sim G$
 - ▶ CCS processes (*Larsen, Xinxin, Parrow, ...*)
 - ▶ Languages (*Petrenko, Yevtushenko, Sangiovanni-Vincentelli, ...*)
 - ▶ Automata, Mealy machines (*von Bochmann, Merlin, ...*)
- Behavioral types (*de Alfaro, Henzinger, ...*)
- Controller synthesis
- Quotient of mu-calculus formulas (*Arnold, Vincent, Walukiewicz*) :

$$P \models \psi/\varphi \Leftrightarrow \exists C. [C \models \varphi \wedge P \times C \models \psi]$$

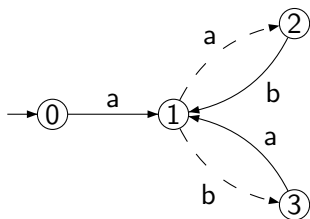
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$$P \models \neg((\neg\psi)/\varphi) \Leftrightarrow \forall C. [C \models \varphi \Rightarrow P \times C \models \psi]$$

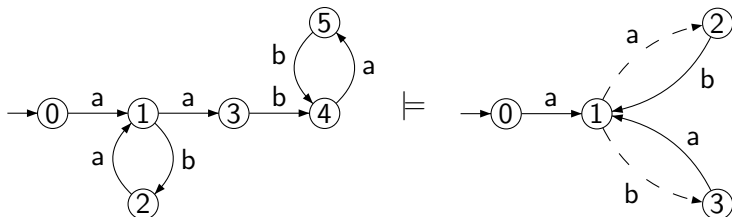
Modal automata (MA) as specifications

- $\mathcal{S} = (Q, q^0, \Delta, may, must)$ with $may, must : Q \rightarrow \mathcal{P}(\Sigma)$
- $must(q) \subseteq may(q)$
- example :



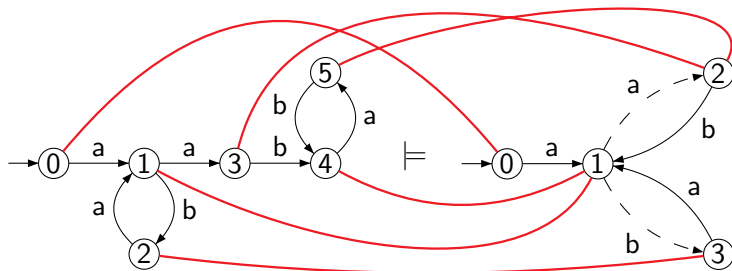
The satisfaction relation \models

- $\mathcal{C} = (R, r^0, \delta)$ (with $out(r) := \{a \in \Sigma. \delta(r, a) \text{ defined}\}$)
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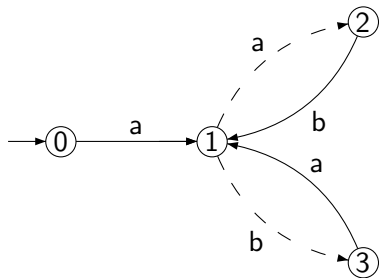


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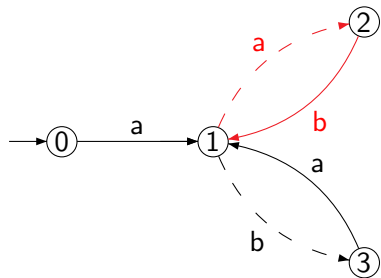


How to build a model of a MA



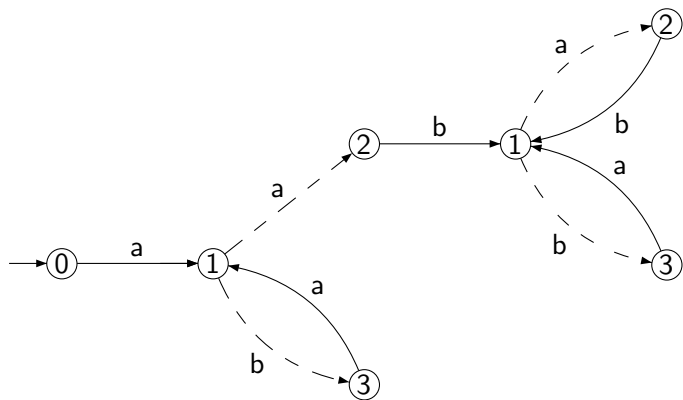
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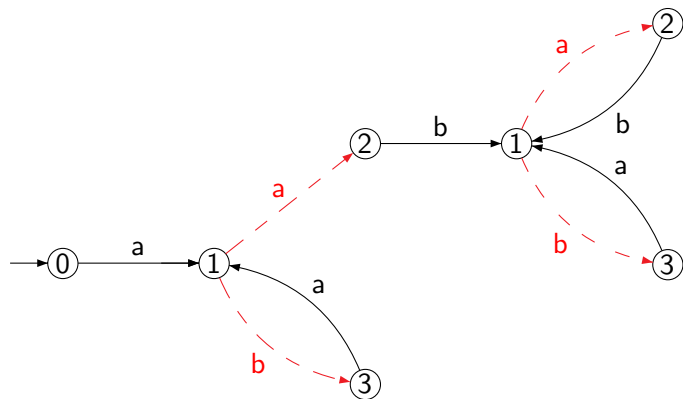
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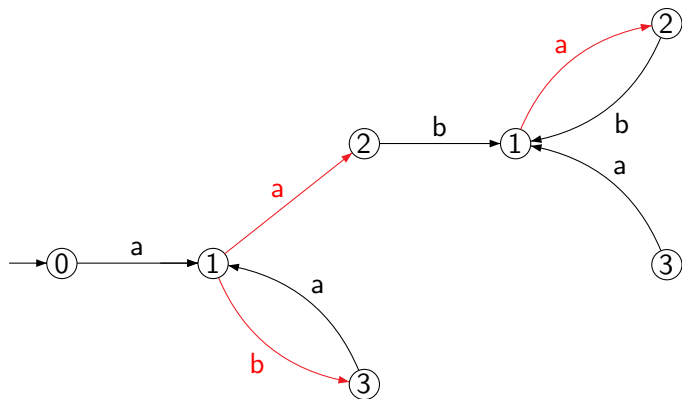
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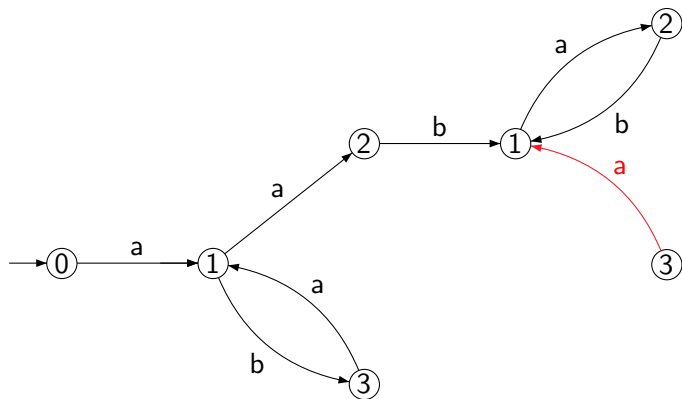
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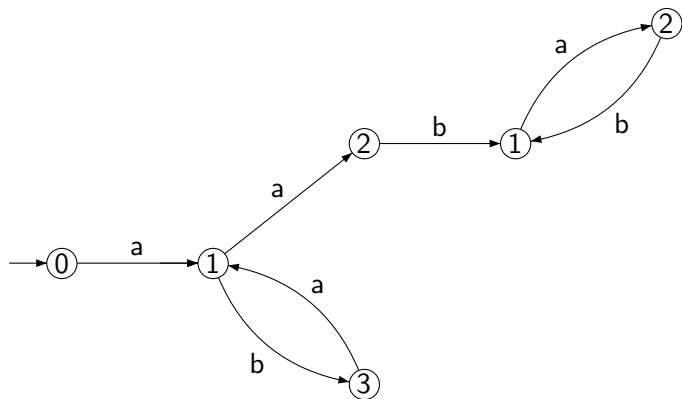
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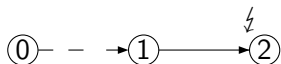
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Unfold, cut (or not) and clean

Pseudo modal automata (pMA)

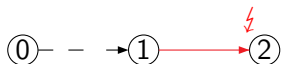
- To represent inconsistency, we let $must(q) \not\subseteq may(q)$ be possible ie. an event can both be required and forbidden
- Reduction $(\cdot)^\downarrow : pMA \rightarrow MA$ such that : $\mathcal{C} \models \mathcal{S} \Leftrightarrow \mathcal{C} \models (\mathcal{S})^\downarrow$



A state with inconsistency can't belong to a simulation relation stating \models (ie. be a state s.t. $must(q) \subseteq out(r) \subseteq may(q)$).

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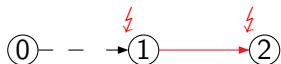
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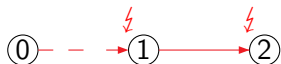
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①

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Two particular pseudo modal automata

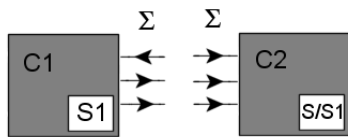
- $\mathcal{S}_{\top} = (\{\top\}, \top, \Delta, \text{may}, \text{must})$ with $\text{must}(\top) = \emptyset$ and $\text{may}(\top) = \Sigma$ and $\Delta(\top, a) = \top$ for all $a \in \Sigma$



- $\mathcal{S}_{\perp} = (\{\perp\}, \perp, \Delta, \text{may}, \text{must})$ with $\text{must}(\perp) = \Sigma$ and $\text{may}(\perp) = \emptyset$ and $\Delta(\perp, a)$ undefined for all $a \in \Sigma$



Synchronous product of implementations



- $\mathcal{C}_1 \times \mathcal{C}_2$ recognizes the language $\mathcal{L}(\mathcal{C}_1) \cap \mathcal{L}(\mathcal{C}_2)$.
- Given $\mathcal{C}_1 = (R_1, r_1^0, \delta_1)$ and $\mathcal{C}_2 = (R_2, r_2^0, \delta_2)$:
 $\mathcal{C}_1 \times \mathcal{C}_2 = (R_1 \times R_2, (r_1^0, r_2^0), \delta)$ with:

$$\delta((r_1, r_2), a) = (r'_1, r'_2) \text{ iff } \begin{cases} \delta_1(r_1, a) = r'_1 \\ \delta_2(r_2, a) = r'_2 \end{cases}$$

Quotient of modal automata

- Given $\mathcal{S} = (Q, q^0, \Delta, \text{may}, \text{must})$ and $\mathcal{S}_1 = (Q_1, q_1^0, \Delta_1, \text{may}_1, \text{must}_1)$
- $\mathcal{S}/\mathcal{S}_1 = ((Q \times Q_1) \cup \{\perp, \top\}, (q^0, q_1^0), \Delta_{/}, \text{may}_{/}, \text{must}_{/})$

\mathcal{S} (global spec.)	\mathcal{S}_1 (reused part)	$\mathcal{S}/\mathcal{S}_1$ (residual spec.)
$q \xrightarrow{a} q'$	$q_1 \xrightarrow{a} q'_1$	$(q, q_1) \xrightarrow{a} (q', q'_1)$
$q \xrightarrow{a} q'$	$(q_1 \dashrightarrow^a q'_1 \text{ or } q_1 \xrightarrow{a})$	$(q, q_1) \xrightarrow{a} \perp$
$q \dashrightarrow^a$	$(q_1 \xrightarrow{a} q'_1 \text{ or } q_1 \dashrightarrow^a)$	$(q, q_1) \dashrightarrow^a$
$q \dashrightarrow^a$	$q_1 \dashrightarrow^a$	$(q, q_1) \dashrightarrow^a \top$
$q \dashrightarrow^a q'$	$q_1 \dashrightarrow^a$	$(q, q_1) \dashrightarrow^a \top$
$q \dashrightarrow^a q'$	$(q_1 \xrightarrow{a} q'_1 \text{ or } q_1 \dashrightarrow^a)$	$(q, q_1) \dashrightarrow^a (q', q'_1)$

Proposition :

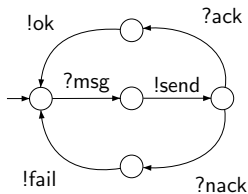
$$\mathcal{C}_2 \models \mathcal{S}/\mathcal{S}_1 \Leftrightarrow \forall \mathcal{C}_1. [\mathcal{C}_1 \models \mathcal{S}_1 \Rightarrow \mathcal{C}_1 \times \mathcal{C}_2 \models \mathcal{S}]$$

- Complexity : $\mathcal{O}(|\Sigma| \times |\mathcal{S}| \times |\mathcal{S}_1|)$

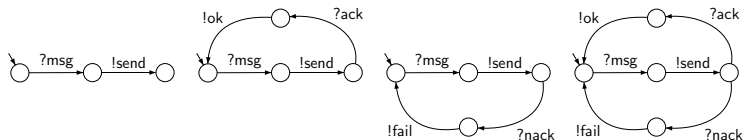
Expressivity of modal automata

- Logical fragment equivalent : conjunctive ν -calculus
- Limit of expressivity : local liveness

Example:



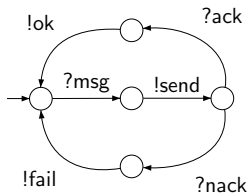
Every message sent must be acknowledged either positively or negatively.



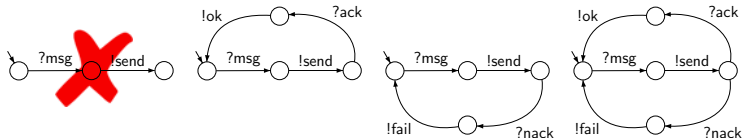
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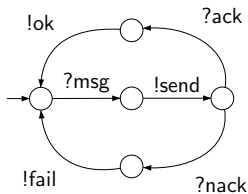
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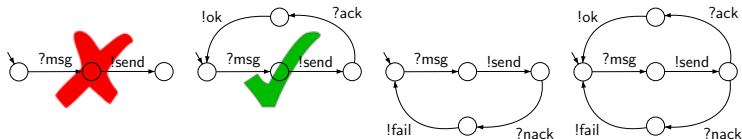
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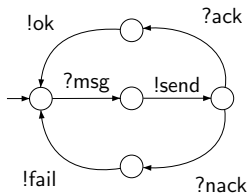
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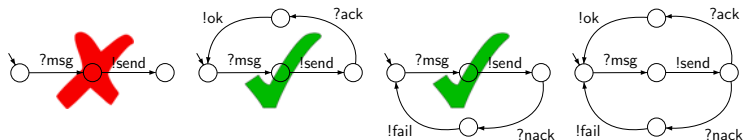
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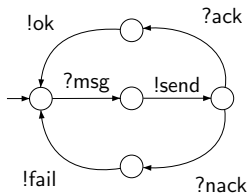
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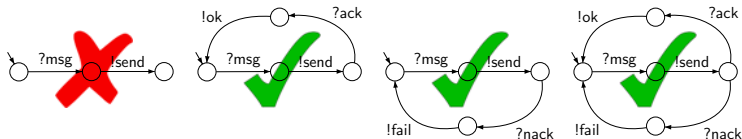
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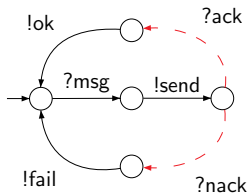
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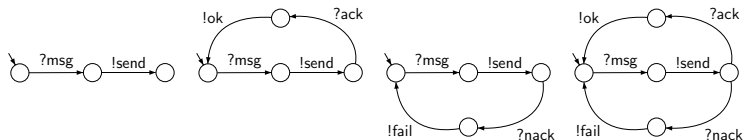
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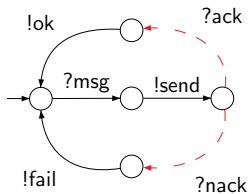
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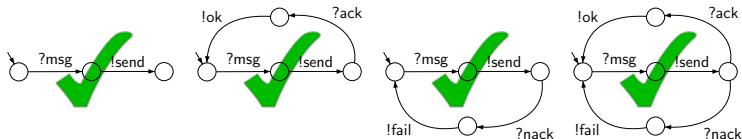
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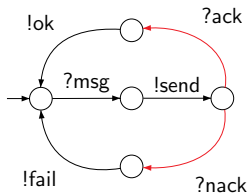
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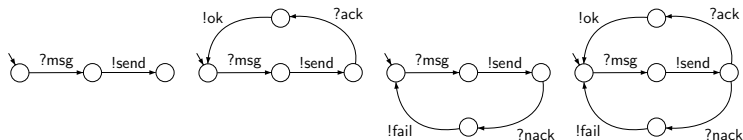
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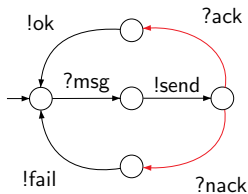
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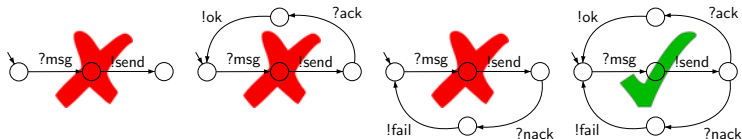
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- When $may(q) = \{ack, nack\}$ and $must(q) = \emptyset$, we have:

$$Acc(q) = \{\emptyset, \{ack\}, \{nack\}, \{ack, nack\}\}$$

We would like to specify:

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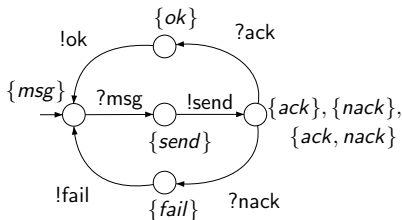
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- We remove the closure under intersection
(Hennessy - *Acceptance Trees*)
 \Rightarrow acceptance automata

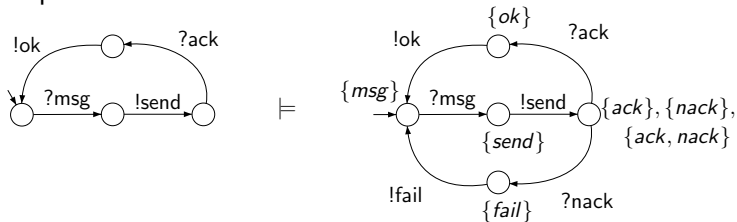
Acceptance automata (AA) as specifications

- $\mathcal{S} = (Q, q^0, \Delta, Acc)$ with $Acc : Q \rightarrow \wp(\wp(\Sigma))$
- Example:



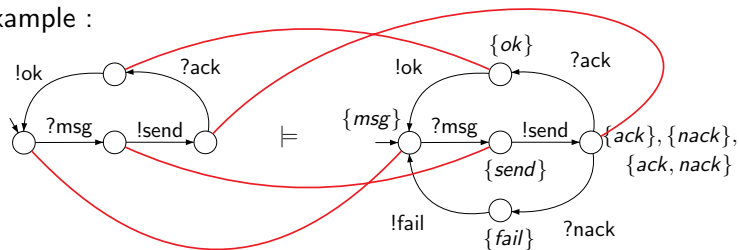
The satisfaction relation \models

- $\mathcal{C} = (R, r^0, \delta)$ (with $out(r) := \{a \in \Sigma. \delta(r, a) \text{ defined}\}$)
- $out(r) \in Acc(q)$
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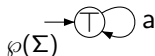


Pseudo acceptance automata (pAA)

- To represent inconsistency, we let $Acc(q) = \emptyset$ be possible
- A state with incoherently specified events can't belong to a simulation relation stating \models (ie. be a state s.t. $out(r) \in Acc(q)$)
- Reduction $(\cdot)^\downarrow : pAA \rightarrow AA$ such that : $\mathcal{C} \models \mathcal{S} \Leftrightarrow \mathcal{C} \models (\mathcal{S})^\downarrow$.

Two particular pAA

- $\mathcal{S}_\top = (\{\top\}, \top, \Delta, \text{Acc})$ with $\text{Acc}(\top) = \wp(\Sigma)$ and $\Delta(\top, a) = \top$ for all $a \in \Sigma$



- $\mathcal{S}_\perp = (\{\perp\}, \perp, \Delta, \text{Acc})$ with $\text{Acc}(\perp) = \emptyset$ and $\Delta(\perp, a)$ undefined for all $a \in \Sigma$



Quotient of acceptance automata

- Given : $\mathcal{S} = (Q, q^0, \Delta, Acc)$ and $\mathcal{S}_1 = (Q_1, q_1^0, \Delta_1, Acc_1)$
- $\mathcal{S}/\mathcal{S}_1 = ((Q \times Q_1) \cup \{\top\}, (q^0, q_1^0), \Delta/, Acc/)$
 - ▶ $Acc/((q, q_1)) =$
 $\{Y \in \wp(\Sigma) \text{ st. } \forall X \in Acc_1(q_1), X \cap Y \in Acc(q)\}$
 - ▶ For all $a \in Y$ where $Y \in Acc/((q, q_1))$:

$$\Delta/((q, q_1), a) = \begin{cases} (q', q_1') & \text{si } \Delta(q, a) = q' \text{ et } \Delta_1(q_1, a) = q_1' \\ \top & \text{sinon} \end{cases}$$

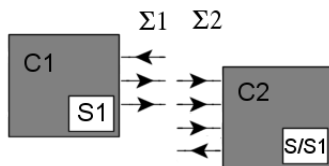
Proposition :

$$\mathcal{C}_2 \models \mathcal{S}/\mathcal{S}_1 \Leftrightarrow \forall \mathcal{C}_1. [\mathcal{C}_1 \models \mathcal{S}_1 \Rightarrow \mathcal{C}_1 \times \mathcal{C}_2 \models \mathcal{S}]$$

- Complexity : $\mathcal{O}(2^{(|\Sigma|+1)} \times |\mathcal{S}| \times |\mathcal{S}_1|)$

Mixed product as composition operation

- Mixed product : $C_1 \sqcap C_2$



Proposition :

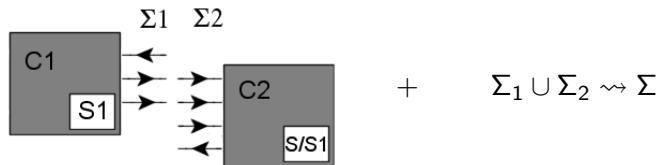
Given $\mathcal{S}(\Sigma)$, $\mathcal{S}_1(\Sigma_1)$ and Σ_2 such that $\Sigma_1 \cup \Sigma_2 = \Sigma$:

$$C_2 \models \mathcal{S} /_{\sqcap} \mathcal{S}_1 \Leftrightarrow \forall C_1. [C_1 \models \mathcal{S}_1 \Rightarrow C_1 \sqcap C_2 \models \mathcal{S}]$$

with $\mathcal{S} /_{\sqcap} \mathcal{S}_1 = \Pi_{\Sigma_2}(\mathcal{S} /_{\tau_{\Sigma}}(\mathcal{S}_1))$.

Internalization of events in composition

- Mixed product + projection : $(C_1 \sqcap C_2) \downarrow_{\Sigma}$



Proposition :

Given $\mathcal{S}(\Sigma)$, $\mathcal{S}_1(\Sigma_1)$ and Σ_2 such that $\Sigma \subseteq \Sigma_1 \cup \Sigma_2$:

$$C_2 \models \mathcal{S} / \sqcap \downarrow \mathcal{S}_1 \Leftrightarrow \forall C_1. [C_1 \models \mathcal{S}_1 \Rightarrow (C_1 \sqcap C_2) \downarrow_{\Sigma} \models \mathcal{S}]$$

with $\mathcal{S} / \sqcap \downarrow \mathcal{S}_1 = \Pi_{\Sigma_2}(\mathcal{S}_{\uparrow(\Sigma_1 \cup \Sigma_2)} / \tau_{\Sigma_1 \cup \Sigma_2}(\mathcal{S}_1))$.

Interactions between \mathcal{C}_1 and \mathcal{C}_2

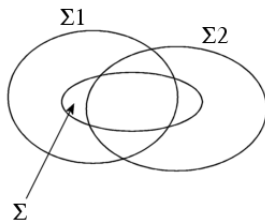
Proposition :

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- $\Sigma_1 \cap \Sigma_2$: control



Interactions between \mathcal{C}_1 and \mathcal{C}_2

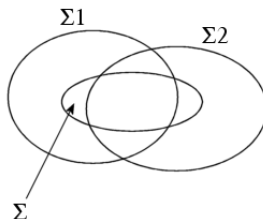
Proposition :

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- $\Sigma_1 \cap \Sigma_2$: control
- $\Sigma \setminus \Sigma_1$: add behaviors



Interactions between \mathcal{C}_1 and \mathcal{C}_2

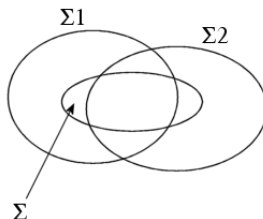
Proposition :

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with $\mathcal{S} /_{\sqcap \downarrow} \mathcal{S}_1 = \Pi_{\Sigma_2}(\mathcal{S}_{\uparrow(\Sigma_1 \cup \Sigma_2)} / \tau_{\Sigma_1 \cup \Sigma_2}(\mathcal{S}_1))$.

- $\Sigma_1 \cap \Sigma_2$: control
- $\Sigma \setminus \Sigma_1$: add behaviors
- $\Sigma_1 \setminus \Sigma$: hide events



Applications (1/2)

- Behavioral reuse

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 - ▶ If $\mathcal{S}/\mathcal{S}_1 = \mathcal{S}_\perp$ then 😞 else 😊
 - ▶ Optimize reuse with greatest model

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- ▶ If $\mathcal{S}/\mathcal{S}_1 = \mathcal{S}_\perp$ then 😞 else 😊
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- Protocol converter synthesis

- ▶ Given $(\mathcal{C}_1, \mathcal{S}_1)$, $(\mathcal{C}_2, \mathcal{S}_2)$ and \mathcal{S} a safety/liveness property
- ▶ Compute $\mathcal{S}/(\mathcal{S}_1 \otimes \mathcal{S}_2)$
- ▶ $\text{Adapt} \models \mathcal{S}/(\mathcal{S}_1 \otimes \mathcal{S}_2) \Rightarrow \mathcal{C}_1 \times \mathcal{C}_2 \times \text{Adapt} \models \mathcal{S}$



Applications (2/2)

- Contract based design

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- Contract based design

- ▶ Contract = a pair of specifications (A, G)
- ▶ Satisfaction of a contract :

$$\mathcal{C} \models G/A \Leftrightarrow \forall \mathcal{E}. [\mathcal{E} \models A \Rightarrow \mathcal{C} \times \mathcal{E} \models G]$$

Future works

- Generalization to a family of specification formalism
- Generalization to an abstract product
- Implementation

Thanks for your attention

Questions ?